

**AN INVENTORY MODEL FOR TIME VARYING HOLDING
COST AND PRICE DISCOUNT ON BACKORDERS IN STOCK
DEPENDENT DEMAND ENVIRONMENT**

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Abstract: In this paper, an inventory model for deteriorating items is studied with stock dependent demand rate where holding cost is expressed as linearly increasing function of time. The study includes some features that are likely to be associated with certain types of inventory in real life, like inventory of seasonal fruits and vegetables. When shortage occurs, only a fraction of unmet demand is backlogged, and the inventory manager offers a discount on it. Conditions of permissible delay in payments are also taken into account. The optimum ordering policy and the optimum discount offered for each backorder are determined by minimizing the total cost in a replenishment interval. Sensitivity analysis is also performed by changing (increasing or decreasing) the parameters and taking one parameter at a time, keeping the remaining parameters at their original values.

MIS 2000 Subject Classification: 90B05.

Key words: Time varying holding cost; deteriorating items; stock dependent demand; price discount on backorder; delay in payment.

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1. Introduction

For certain types of inventory, particularly consumer goods in supermarkets, demand are greatly influenced by the stock level. An increase in shelf space for an item attracts the consumers to buy more owing to its visibility, popularity or variety. Giri and Chaudhuri (1998) established an EOQ model for perishable product where demand rate is a function of on hand inventory. Datta and Paul (2001) analysed a multi-period EOQ model with stock dependent and price sensitive demand rate. Vishnoi et al. (2010) determined the optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and permissible delay in payments under limited storage capacity.

In most inventory models, it is assumed that holding cost is constant. But, in case of seasonal fruits and vegetables, the longer these items are kept in storage, the more sophisticated the storage facilities and services needed, and therefore, the higher the holding cost. The variability in the holding cost was first introduced by Muhlemann and Valtis-Spanopoulous (1980). Alfares (2007) presented an inventory system with stock dependent demand, in which the holding cost is a step function of storage time. Roy (2008) developed an inventory model for deteriorating items by considering demand rate and holding cost as linear functions of time.

In classical inventory models with shortages, it is generally assumed that the unmet demand is either completely lost or completely backlogged. In cases of many products of famous brands or fashionable commodities, such as hi-fi equipment, trendy apparel, cosmetics and clothes, customers prefer their demands to be backordered. Some customers may be willing to wait till the stock is replenished, while some may be impatient and satisfy their demand immediately from some other source. In some situations, the inventory manager may offer a discount on backorders and/or reduction in waiting time to tempt customers to wait. Ouyang et al. (2003) developed a periodic review inventory model with backorder discounts to accommodate more practical features of the real inventory systems. Chuang et al. (2004) discussed a distribution free procedure for mixed inventory model with backorder discount and variable lead time. Pentico & Drake (2011) modified the existing model for the deterministic economic order quantity with partial backordering by making the backordering percentage a function of the size of the discount. Pal and Chandra (2012) studied a deterministic inventory model with shortages. They

considered only a fraction of the unmet demand is backlogged, and the inventory manager offers a discount on it. Pal and Chandra (2014) developed a periodic review inventory model with stock dependent demand, permissible delay in payment and price discount on backorders.

In many real-life situations, the supplier allows the inventory manager a certain fixed period of time to settle his accounts. No interest is charged during this period, but beyond it the manager has to pay an interest to the supplier. During the permitted time period, the manager is free to sell his goods, accumulate revenue and earn interest. Goyal (1985) is the pioneer researcher who formed inventory models taking the condition of permissible delay in payments. Kumar et al. (2009) developed an inventory model with power demand rate, incremental holding cost and permissible delay in payments. Misra et al. (2011) derived an optimal inventory replenishment policy for two parameters Weibull deteriorating items with a permissible delay in payment under inflation over the finite planning horizon.

In this paper, an inventory model for deteriorating items is developed where holding cost is expressed as linearly increasing function of time and demand rate is dependent on the inventory level. The supplier allows the inventory manager a fixed time interval to settle his dues and the inventory manager offers his customer a discount in case he is willing to backorder his demand when there is a stock-out. The paper is organized as follows. Assumptions and notations are presented in Section 2. In Section 3, the model is formulated and the optimal order quantity and backorder price discount determined. In Section 4, numerical examples are cited to illustrate the policy and to analyze the sensitivity of the model with respect to the model parameters. Concluding remarks are given in Section 5.

2. Notations and Assumptions

To develop the model, the following notations and assumptions have been used.

Notations

$I(t)$ = inventory level at time point t

b = fraction of the demand backordered during stock out

b_0 = upper bound of backorder ratio

K = ordering cost per order

P = purchase cost per unit

s_1 = backorder cost per unit backordered per unit time

s_2 = cost of a lost sale

θ = rate of deterioration

π = price discount on unit backorder offered

π_0 = marginal profit per unit

I_e = interest that can be earned per unit time

I_r = interest payable per unit time beyond the permissible delay period ($I_r > I_e$)

M = permissible delay in settling the accounts, $0 < M < T$

T = length of a replenishment cycle

T_1 = time taken for stock on hand to be exhausted, $0 < T_1 < T$

S = maximum stock height in a replenishment cycle

s = shortage at the end of a replenishment cycle

Assumptions

1. The model considers only one item in inventory.
2. Replenishment of inventory occurs instantaneously on ordering, that is, lead time is zero.
3. Shortages are allowed, and a fraction b of unmet demands during stock-out is backlogged.
4. During the stock-out period, the backorder fraction b is directly proportional to the price discount π offered by the inventory manager. Thus,

$$b = \frac{b_0}{\pi_0} \pi, \text{ where } 0 \leq b_0 \leq 1, 0 \leq \pi \leq \pi_0$$

5. Demand rate $R(t)$ at time t is

$$R(t) = \alpha + \beta I(t) \quad \text{for } 0 < t < T_1$$

$$= \alpha \quad \text{for } T_1 < t < T$$

where α = fixed demand per unit time, $\alpha > 0$, β = fraction of total inventory demanded per unit time under the influence of stock on hand, $0 < \beta < 1$.

6. Holding cost $h(t)$ per item per unit time is assumed to be time dependent

$$h(t) = \gamma + \delta t \quad \text{where } \gamma > 0, \delta > 0$$

3. Model Formulation

The planning period is divided into reorder intervals, each of length T units. Orders are placed at time points $0, T, 2T, 3T, \dots$, the order quantity being just sufficient to bring the stock height to a certain maximum level S .

Depletion of inventory occurs due to demand and deterioration during the period $(0, T_1)$, $T_1 < T$, and in the interval (T_1, T) shortage occurs, of which a fraction b is backlogged. Hence, the variation in inventory level with respect to time is given by

$$\frac{d}{dt}I(t) + \theta I(t) = -\alpha - \beta I(t), \quad \text{if } 0 < t < T_1$$

$$\frac{d}{dt}I(t) = -b\alpha, \quad \text{if } T_1 < t < T$$

Since $I(T_1) = 0$, we get

$$I(t) = \frac{\alpha}{\theta + \beta} \left(e^{(\theta + \beta)(T_1 - t)} - 1 \right), \quad \text{if } 0 < t < T_1$$

$$= b\alpha(T_1 - t), \quad \text{if } T_1 < t < T$$

$$\text{Hence, } S = \frac{\alpha}{\theta + \beta} \left(e^{(\theta + \beta)T_1} - 1 \right)$$

Then,

Ordering cost during a cycle (OC) = K

Holding cost of inventories during a cycle (HC)

$$= \int_0^{T_1} h(t)I(t)dt$$

$$= \frac{\alpha}{\theta + \beta} \left(\frac{1}{\theta + \beta} \left(e^{(\theta + \beta)T_1} - 1 \right) \left(\frac{\delta}{\theta + \beta} + \gamma \right) - T_1 \left(\gamma + \frac{\delta}{\theta + \beta} + \frac{1}{2} \delta T_1 \right) \right)$$

Backorder cost during a cycle (BC)

$$= -s_1 \int_{T_1}^T I(t)dt = \frac{s_1 b \alpha}{2} (T - T_1)^2$$

Lost sales cost during a cycle (LC) = $s_2(1 - b)\alpha(T - T_1)$

As regards the permissible delay in payment, there can be two possibilities: $M \leq T_1$ and $M > T_1$.

Case 1: $M \leq T_1$

For $M \leq T_1$, the inventory manager has stock on hand beyond M , and so he can use the sale revenue to earn interest at a rate I_e during $(0, T_1)$. The interest earned by the inventory manager is, therefore,

$$IE_1 = PI_e \int_0^{T_1} I(t) dt = \frac{PI_e \alpha}{\theta + \beta} \left(\frac{1}{\theta + \beta} (e^{(\theta + \beta)T_1} - 1) - T_1 \right)$$

Beyond the fixed settlement period, the unsold stock is financed with an interest rate I_r , so that the interest payable by the inventory manager is

$$IP_1 = PI_r \int_M^{T_1} I(t) dt = \frac{PI_r \alpha}{\theta + \beta} \left(\frac{1}{\theta + \beta} (e^{(\theta + \beta)(T_1 - M)} - 1) - (T_1 - M) \right)$$

Hence, the cost per unit length of a replenishment cycle is given by

$$\begin{aligned} C_1(T_1, T, b) &= \frac{1}{T} [\text{OC} + \text{HC} + \text{BC} + \text{LC} + \text{IP}_1 - \text{IE}_1] \\ &= \frac{1}{T} \left[K + \frac{\alpha}{\theta + \beta} \left(\frac{1}{\theta + \beta} (e^{(\theta + \beta)T_1} - 1) \left(\frac{\delta}{\theta + \beta} + \gamma \right) - T_1 \left(\gamma + \frac{\delta}{\theta + \beta} + \frac{1}{2} \delta T_1 \right) \right) \right. \\ &\quad \left. + \frac{s_1 b \alpha}{2} (T - T_1)^2 + s_2 (1 - b) \alpha (T - T_1) \right. \\ &\quad \left. + \frac{PI_r \alpha}{\theta + \beta} \left(\frac{1}{\theta + \beta} (e^{(\theta + \beta)(T_1 - M)} - 1) - (T_1 - M) \right) - \frac{PI_e \alpha}{\theta + \beta} \left(\frac{1}{\theta + \beta} (e^{(\theta + \beta)T_1} - 1) - T_1 \right) \right] \\ &= \frac{N_1(T_1, T, b)}{T} \end{aligned}$$

The optimal values of T_1 , T and b , which minimize $C_1(T_1, T, b)$, must satisfy the following equations:

$$\begin{aligned} \frac{\alpha}{\theta + \beta} \left(\left(\frac{\delta}{\theta + \beta} + \gamma \right) e^{(\theta + \beta)T_1} - \left(\gamma + \frac{\delta}{\theta + \beta} + \delta T_1 \right) \right) + \frac{P\alpha}{\theta + \beta} e^{(\theta + \beta)T_1} (I_r e^{-(\theta + \beta)M} - I_e) \\ = s_1 b \alpha (T - T_1) + s_2 (1 - b) \alpha + \frac{P\alpha}{\theta + \beta} (I_r - I_e) \end{aligned} \quad (3.1)$$

$$s_1 b \alpha (T - T_1) + s_2 (1 - b) \alpha = C_1(T_1, T, b) \quad (3.2)$$

$$T - T_1 = \frac{2s_2}{s_1} \quad (3.3)$$

Case 2: $M > T_1$

Since $M > T_1$, the inventory manager pays no interest, but earns interest in the interval $(0, M)$ at a rate I_e .

The interest earned by the inventory manager is given by

$$IE_2 = PI_e \int_0^M I(t) dt = PI_e \alpha \left(\frac{1}{(\theta + \beta)^2} (e^{(\theta + \beta)T_1} - 1) - \frac{T_1}{\theta + \beta} + \frac{b}{2} (M - T_1)^2 \right)$$

Hence, the cost per unit length of a replenishment cycle is given by

$$\begin{aligned} C_2(T_1, T, b) &= \frac{1}{T} [\text{OC} + \text{HC} + \text{BC} + \text{LC} - IE_2] \\ &= \frac{1}{T} \left(K + \frac{\alpha}{\theta + \beta} \left(\frac{1}{\theta + \beta} (e^{(\theta + \beta)T_1} - 1) \left(\frac{\delta}{\theta + \beta} + \gamma \right) - T_1 \left(\gamma + \frac{\delta}{\theta + \beta} + \frac{1}{2} \delta T_1 \right) \right) \right. \\ &\quad \left. + \frac{s_1 b \alpha}{2} (T - T_1)^2 + s_2 (1 - b) \alpha (T - T_1) \right. \\ &\quad \left. - PI_e \alpha \left(\frac{1}{(\theta + \beta)^2} (e^{(\theta + \beta)T_1} - 1) - \frac{T_1}{\theta + \beta} + \frac{b}{2} (M - T_1)^2 \right) \right) \\ &= \frac{N_2(T_1, T, b)}{T} \end{aligned}$$

The optimal values of T_1 , T and b , which minimize $C_2(T_1, T, b)$, must satisfy the following equations:

$$\begin{aligned} \frac{\alpha}{\theta + \beta} \left(\left(\frac{\delta}{\theta + \beta} + \gamma \right) e^{(\theta + \beta)T_1} - \left(\gamma + \frac{\delta}{\theta + \beta} + \delta T_1 \right) \right) \\ = s_1 b \alpha (T - T_1) + s_2 (1 - b) \alpha + PI_e \left(\frac{\alpha}{\theta + \beta} (e^{(\theta + \beta)T_1} - 1) - b \alpha (M - T_1) \right) \end{aligned} \quad (3.4)$$

$$s_1 b \alpha (T - T_1) + s_2 (1 - b) \alpha = C_2(T_1, T, b) \quad (3.5)$$

$$T - T_1 = \frac{2s_2}{s_1} \quad (3.6)$$

The total expected cost per unit length of a replenishment cycle is, therefore, given by

$$\begin{aligned} C(T_1, T, b) &= C_1(T_1, T, b) \quad \text{if } T_1 \geq M \\ &= C_2(T_1, T, b) \quad \text{if } T_1 < M \end{aligned}$$

The optimal values of the decision variables (T_1, T, b) minimizing $C(T_1, T, b)$ will be the set of values minimizing $C_1(T_1, T, b)$ if $\min C_1(T_1, T, b) \leq \min C_2(T_1, T, b)$, or the set of values minimizing $C_2(T_1, T, b)$ if $\min C_2(T_1, T, b) \leq \min C_1(T_1, T, b)$.

4. Numerical Illustration and Sensitivity Analysis

Since it is difficult to find closed form solutions to the sets of equations (3.1) - (3.3) and (3.4) - (3.6), we numerically find solutions to the equations for given sets of model parameters using the statistical software MATLAB. The following tables show the change in optimal inventory policy with change in a model parameter, when the other parameters remain fixed.

Table 1: Showing the optimal inventory policy for different values of γ , when $K=500$, $P = 15$, $I_r=0.5$, $I_e=0.3$, $\alpha = 50$, $\beta = 0.5$, $M = 2$, $\theta = 0.4$, $\delta = 0.6$, $s_1=60$ and $s_2=70$.

γ	T_I	T	b	$C(T_I, T, b)$
3	4.8446	7.1779	0.8572	1235.25
4	3.6819	6.0153	0.8939	1599.09
5	3.1980	5.5313	0.9834	1789.12
8	2.4987	4.8320	0.9958	2114.82
12	2.0478	4.3812	0.9969	2361.90
15	1.7957	4.1290	1.0000	2488.37

Table 2: Showing the optimal inventory policy for different values of θ , when $K=500$, $P = 15$, $I_r=0.5$, $I_e=0.3$, $\alpha = 50$, $\beta = 0.5$, $M = 2$, $\gamma = 5$, $\delta = 0.6$, $s_1=60$ and $s_2=70$.

θ	T_I	T	b	$C(T_I, T, b)$
0.1	3.5534	5.8868	0.9393	1713.71
0.2	3.4250	5.7583	0.9537	1739.03
0.3	3.3072	5.6406	0.9682	1764.12
0.35	3.2516	5.5850	0.9757	1776.62
0.4	3.1980	5.5313	0.9834	1789.12
0.43	3.1666	5.4999	0.9883	1796.63

Table 3: Showing the optimal inventory policy for different values of I_r , when $K=500$, $P = 15$, $I_e=0.3$, $\alpha = 50$, $\beta = 0.5$, $M = 2$, $\theta = 0.4$, $\gamma = 5$, $\delta = 0.6$, $s_1=60$ and $s_2=70$.

I_r	T_I	T	b	$C(T_I, T, b)$
0.4	3.2729	5.6062	0.9741	1773.71
0.5	3.1980	5.5313	0.9834	1789.12

0.6	3.1324	5.4657	0.6513	1802.67
0.7	3.0744	5.4077	0.9820	1814.69
0.8	3.0226	5.3559	0.9632	1825.45
0.9	2.9760	5.3094	0.5800	1835.14

Table 4: Showing the optimal inventory policy for different values of I_e , when $K=500$, $P = 15$, $I_r= 0.5$, $\alpha = 50$, $\beta = 0.5$, $M = 2$, $\theta = 0.4$, $\gamma = 5$, $\delta = 0.6$, $s_1= 60$ and $s_2= 70$.

I_e	T_1	T	b	$C(T_1, T, b)$
0.1	2.4987	4.8320	0.9958	2114.82
0.2	2.7744	5.1077	0.3336	1978.54
0.3	3.1980	5.5313	0.9834	1789.12
0.35	3.5331	5.8664	0.9295	1654.76
0.4	4.0871	6.4205	0.8780	1458.88
0.45	5.7702	8.1036	0.8391	1015.64

Table 5: Showing the optimal inventory policy for different values of s_1 , when $K=500$, $P = 15$, $I_r= 0.5$, $I_e= 0.3$, $\alpha = 50$, $\beta = 0.5$, $M = 2$, $\theta = 0.4$, $\gamma = 5$, $\delta = 0.6$ and $s_2= 70$.

s_1	T_1	T	b	$C(T_1, T, b)$
40	3.3298	6.8298	0.9337	2084.17
50	3.2591	6.0591	0.9666	1921.55
60	3.1980	5.5313	0.9834	1789.12
70	3.1443	5.1443	0.9836	1678.84
80	3.0967	4.8467	0.9824	1585.37

Table 6: Showing the optimal inventory policy for different values of s_2 , when $K=500$, $P = 15$, $I_r= 0.5$, $I_e= 0.3$, $\alpha = 50$, $\beta = 0.5$, $M = 2$, $\theta = 0.4$, $\gamma = 5$, $\delta = 0.6$ and $s_1= 60$.

s_2	T_1	T	b	$C(T_1, T, b)$
40	2.6833	4.0166	0.9777	924.95
50	2.8703	4.5370	0.9734	1192.97

60	3.0417	5.0417	0.3333	1482.53
70	3.1980	5.5313	0.9834	1789.12
80	3.3405	6.0072	0.9604	2109.74
90	3.4710	6.4710	0.9323	2442.20

Table 7: Showing the optimal inventory policy for different values of M , when $K=500$, $P = 15$, $I_r= 0.5$, $I_e= 0.3$, $\alpha = 50$, $\beta = 0.5$, $\theta = 0.4$, $\gamma= 5$, $\delta = 0.6$, $s_1= 60$ and $s_2= 70$.

M	T_1	T	b	$C(T_1, T, b)$
1	2.6629	4.9963	0.3333	2002.91
1.5	2.9497	5.2831	0.9856	1878.17
2	3.1980	5.5313	0.9834	1789.12
2.5	3.4030	5.7363	0.9774	1730.28
3	3.5650	5.8984	0.9895	1696.25
4	3.6916	6.0250	1.0000	1678.76

The above tables show that, for other parameters remaining constant,

- both T_1 and T are decreasing in γ , θ, I_r and s_1 but increase as I_e , s_2 and M increase;
- b , and hence π , decreases with increase in s_2 , but increases with γ , θ , s_1 and M ;
- the minimum cost per unit length of a reorder interval increases as γ , θ , I_r and s_2 increase, but decreases with increase in I_e , s_1 and M .

The above observations indicate that, with a view to minimizing total cost, the policy should be to maintain high inventory level for low backorder cost but high lost sales cost. Also, higher the permissible delay period higher should be the price discount offered on a backorder.

Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 5% and 10% and taking one parameter at a time, keeping the remaining parameters at their original values. Let us consider the following model parameters: $K=500$, $P = 15$, $I_r= 0.5$, $I_e= 0.3$, $\alpha = 50$, $\beta = 0.5$, $M = 2$, $\theta = 0.4$, $\gamma= 5$, $\delta = 0.6$, $s_1= 60$ and $s_2= 70$. The following table gives the percentage change in the decision variables and total cost over an inventory cycle with change in the model parameters.

Table 8: The results of sensitivity analysis

Parameter	% change	% change in T_1	% change in T	% change in b	% change in $C(T_1, T, b)$
M	-10	-2.95	-1.70	0.49	1.77
	-5	-1.45	-0.84	0.24	0.85
	5	1.39	0.80	-0.18	-0.78
	10	2.73	1.58	-66.15	-1.50
s_1	-10	1.12	5.33	-0.98	4.27
	-5	0.55	2.54	-0.48	2.07
	5	-0.53	-2.31	0.48	-1.96
	10	-1.03	-4.43	-0.27	-3.83
s_2	-10	-3.37	-6.17	-66.11	-12.09
	-5	-1.66	-3.07	-1.27	-6.09
	5	1.61	3.04	-0.84	6.19
	10	3.16	6.05	-1.65	12.47
I_r	-10	1.13	0.65	-0.45	-0.42
	-5	0.56	0.32	-0.22	-0.20
	5	-0.54	-0.31	0.21	0.20
	10	-1.06	-0.61	0.45	0.39
I_e	-10	-4.74	-2.74	-61.00	3.64
	-5	-2.48	-1.43	-1.21	1.88
	5	2.73	1.58	-1.34	-2.03
	10	5.78	3.34	-2.86	-4.23
γ	-10	6.51	3.76	-3.24	-4.75
	-5	3.06	1.77	-1.50	-2.26
	5	-2.74	-1.58	0.06	2.08
	10	-5.21	-3.01	-0.07	4.01
θ	-10	1.34	0.77	-0.64	-0.56
	-5	0.66	0.38	-0.32	-0.28
	5	-0.66	-0.38	0.33	0.28
	10	-1.30	-0.75	-67.88	0.56

5. CONCLUSION

The paper studies an inventory model for deteriorating items with stock dependent demand rate allowing shortages. It is also considered that the holding cost is linearly increasing function of time. The study includes some features that are likely to be associated with certain types of inventory in real life, like inventory of seasonal fruits and vegetables. A fraction of the demand is

backlogged, and the inventory manager offers a discount to each customer who is ready to wait till fulfillment of his demand. The replenishment source allows the inventory manager a certain time period to settle his accounts. No interest is charged during this period, but beyond it the manager has to pay an interest. The optimum ordering policy and the optimum discount offered for each backorder are determined by minimizing the total cost in a replenishment interval. Through numerical study, it is observed that for higher permissible delay period, it is beneficial to the inventory manager to offer the customers high discount on backorders.

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